(31) Evaluate $\underset{x\to 0}{Lt} \sin x \log x$

Ans. → $\underset{x\to 0}{Lt} \sin x \log x \; [0\times\infty]$

$\therefore \underset{x\to 0}{Lt} \dfrac{\log x}{\frac{1}{\sin x}}$

$= \underset{x\to 0}{Lt} \dfrac{\log x}{\operatorname{cosec} x} \; \left[\dfrac{\infty}{\infty}\right]$

$= \underset{x\to 0}{Lt} \dfrac{\frac{1}{x}}{-\cot x \cdot \operatorname{cosec} x}$

$= \underset{x\to 0}{Lt} - \dfrac{\frac{1}{x}}{\frac{\cos x}{\sin x} \times \frac{1}{\sin x}} = \underset{x\to 0}{Lt} - \dfrac{1}{x} \times \dfrac{\sin x}{\cos x} \times \dfrac{\sin x}{1}$

$= \underset{x\to 0}{Lt} - \dfrac{\tan x \cdot \sin x}{x} \; \left[\dfrac{0}{0}\right]$

Hence from L' Hospital's Rule

$= \underset{x\to 0}{Lt} - \dfrac{\tan x \cdot \cos x + \sec^2 x \cdot \sin x}{1} = 0$ Ans

(32) Evaluate $\underset{x\to 1}{Lt} (1-x)\tan\dfrac{\pi x}{2}$.

Ans. → $\underset{x\to 1}{Lt} (1-x)\tan\dfrac{\pi x}{2} \; [0\times\infty]$

$$\therefore \underset{x \to 1}{Lt} \frac{1-x}{\cot \frac{\pi x}{2}} \left[\frac{0}{0}\right]$$

Hence from L' Hospital Rule

$$\underset{x \to 1}{Lt} \frac{0-1}{-\operatorname{cosec}^2 \frac{\pi x}{2} \times \frac{\pi}{2}}$$

$$\underset{x \to 1}{Lt} \frac{\cancel{-}1}{\cancel{-}\frac{\pi}{2} \operatorname{cosec}^2 \frac{\pi x}{2}}$$

$$= \underset{x \to 1}{Lt} \frac{1}{\frac{\pi}{2} \operatorname{cosec}^2 \frac{\pi x}{2}}$$

$$= \underset{x \to 1}{Lt} \frac{2}{\pi} \times \sin^2 \frac{\pi x}{2}$$

$$= \frac{2}{\pi} \times \sin^2 \frac{\pi}{2} \times 1$$

$$= \frac{2}{\pi} \cdot (1)^2$$

$$= \frac{2}{\pi} \text{ Ans.}$$

(33) Evaluate $\underset{x \to a}{Lt} \sqrt{a^2-x^2} \cdot \cot \left[\frac{\pi}{2} \sqrt{\frac{a-x}{a+x}}\right]$

Ans. $\to \underset{x \to a}{Lt} \sqrt{a^2-x^2} \cdot \cot \left[\frac{\pi}{2} \sqrt{\frac{a-x}{a+x}}\right] \quad [0 \times \infty]$

$$= \underset{x \to a}{Lt} \frac{\sqrt{a^2-x^2}}{\tan\left[\frac{\pi}{2}\left(\sqrt{\frac{a-x}{a+x}}\right)\right.} \left[\frac{0}{0}\right]$$

Hence from L' Hospital's Rule, we have

$$= \lim_{x\to a} \frac{\frac{1}{2\sqrt{a^2-x^2}} \times -2x}{\sec^2\left(\frac{\pi}{2}\sqrt{\frac{a-x}{a+x}}\right) \times \frac{\pi}{2}} \times \frac{1}{2\sqrt{\frac{a-x}{a+x}}\left[\frac{(a+x)\times -1-(a-x)}{(a+x)^2}\right]}$$

$\sec x\,|$

$$= \lim_{x\to a} \frac{-x}{\sqrt{a+x}\cdot\sqrt{a-x}} \times \frac{\cos^2\left(\frac{\pi}{2}\sqrt{\frac{a-x}{a+x}}\right) \times (a+x)^2}{\frac{\pi}{4} \times \frac{\sqrt{a+x}}{\sqrt{a-x}} \times -\left[a+\cancel{x}+a-\cancel{x}\right]}$$

$$= \lim_{x\to 0} \cancel{+}\frac{x \times \cos^2\left(\frac{\pi}{2}\sqrt{\frac{a-x}{a+x}}\right) \times (a+x)^{\cancel{2}} \times \cancel{\sqrt{a-x}}}{\sqrt{a+x}\cdot\cancel{\sqrt{a-x}} \times \frac{\pi}{4}\sqrt{a+x} \times \cancel{+}2a}$$

$$= \frac{a \times 1 \times \cancel{2a}}{\frac{\pi}{4} \times \cancel{2a}} = \frac{4a}{\pi}.$$

(34) Evaluate $\lim_{x\to\infty} 2^x \sin\frac{a}{2^x}$.

Ans. → $\lim_{x\to\infty} 2^x \sin\frac{a}{2^x}$ Let $\frac{a}{2^x} = \theta$

$\therefore 2^x = \frac{a}{\theta}$

when $x\to\infty$, then $\theta\to 0$

$$= \lim_{\theta\to 0} \frac{a}{\theta}\sin\theta$$

$$= \lim_{\theta\to 0} \frac{\sin\theta}{\theta}\cdot a$$

$= 1 \times a = a$ Ans.

(35) Evaluate $\underset{x\to\infty}{Lt}\ 2x\tan\frac{a}{2x}$.

Ans: $\underset{x\to\infty}{Lt}\ 2x\tan\frac{a}{2x}\ [\infty\times 0]$

$=\underset{x\to\infty}{Lt}\ a\times\frac{2x}{a}\cdot\tan\frac{a}{2x}$

Let $\frac{a}{2x}=\theta$,

$\therefore \cancel{2x=\frac{a}{\theta}}$

So that $\theta\to 0$ as,

$\theta\to 0$

$=\underset{\theta\to 0}{Lt}\ a\times\frac{1}{\theta}\cdot\tan\theta$

$=\underset{\theta\to 0}{Lt}\ \frac{a}{\theta}\times\frac{\tan\theta}{\theta}$

$= a\times 1 = a$ Ans.

(36) Evaluate $\underset{x\to\frac{\pi}{2}}{Lim}(\sec x-\tan x)$.

Ans. $\to \underset{x\to\frac{\pi}{2}}{Lt}(\sec x-\tan x)\ [\infty-\infty]$

$=\underset{x\to\frac{\pi}{2}}{Lt}\left(\frac{1}{\cos x}-\frac{\sin x}{\cos x}\right)$

$=\underset{x\to\frac{\pi}{2}}{Lt}\left(\frac{1-\sin x}{\cos x}\right)\left[\frac{0}{0}\right]$

Hence from L' Hospital's Rule, we have

$=\underset{x\to\frac{\pi}{2}}{Lt}\ \frac{1-\cos x}{-\sin x}$

$=\frac{0}{1}=0$ Ans.

(37) Evaluate $\underset{x \to 1}{Lt} \left( \frac{x}{x-1} - \frac{1}{\log_e x} \right)$.

Ans. → $\underset{x \to 1}{Lt} \left( \frac{x}{x-1} - \frac{1}{\log_e x} \right) \left[ \infty - \infty \right]$

$= \underset{x \to 1}{Lt} \frac{x \log x - x + 1}{x - 1 \cdot \log x} \left[ \frac{0}{0} \right]$

Hence from L' Hospital's Rule, we have.

$= \underset{x \to 1}{Lt} \frac{1 \times \log x + x \times \frac{1}{x} - 1}{\cancel{x - 1 + \log x} \quad \log x + (x-1) \cdot \frac{1}{x}}$

$= \underset{x \to 1}{Lt} \frac{\log x + \cancel{\frac{x - x}{x}}}{x \log x + \frac{x-1}{x}}$

$= \underset{x \to 1}{Lt} \frac{x \log x}{x \log x + x - 1} \left[ \frac{0}{0} \right]$

Hence from L' Hospital's Rule, we have,

$= \underset{x \to 1}{Lt} \frac{1 \times \log x + x \cdot \frac{1}{x}}{1 \times \log x + x \cdot \frac{1}{x} + 1}$

$= \frac{0 + 1 \times \frac{1}{1}}{0 + 1 \times \frac{1}{1} + 1} = \frac{\frac{1}{1}}{\frac{1+1}{1}} = \frac{\frac{1}{1}}{\frac{2}{1}} = \frac{1}{1} \times \frac{1}{2} = \frac{1}{2}$ Ans

(38) Evaluate $\underset{x \to \frac{\pi}{2}}{Lt} \left( x \tan x - \frac{\pi}{2} \sec x \right)$.

Ans.→ $\lim_{x \to \frac{\pi}{2}} \left(x \tan x - \frac{\pi}{2} \sec x\right)$ $[\infty - \infty]$

$= \lim_{x \to \frac{\pi}{2}} \left(\frac{x \sin x}{\cos x} - \frac{\pi}{2\cos x}\right)$

$= \lim_{x \to \frac{\pi}{2}} \frac{2x \sin x - \pi}{2\cos x}$ $\left[\frac{0}{0}\right]$

Hence from L' Hospital's Rule,

$= \lim_{x \to \frac{\pi}{2}} \frac{2x \cos x + 2 \times 1 \sin x}{-2 \sin x}$

$= \frac{2\frac{\pi}{2} \cdot \cos\frac{\pi}{2} + 2\sin\frac{\pi}{2}}{-2\sin\frac{\pi}{2}} = \frac{0+2}{-2} = \frac{2}{-2}$

$= -1$ Ans

(39) Find the limiting value of $\frac{1}{x} - \cot x$, when $x$ tends to zero.

Ans.→ $\lim_{x \to 0} \frac{1}{x} - \cot x$ $[\infty - \infty]$

$\therefore \lim_{x \to 0} \frac{1}{x} - \frac{\cos x}{\sin x}$

$= \lim_{x \to 0} \frac{\sin x - x\cos x}{x \sin x}$ $\left[\frac{0}{0}\right]$

Hence from L' Hospital's Rule,

$= \lim_{x \to 0} \frac{\cos x - 1 \times \cos x + x \sin x}{1 \times \sin x + x \cos x}$ $\left[\frac{0}{0}\right]$

~~Hence from~~

$$= \lim_{x\to 0} \frac{x\sin x}{\sin x + x\cos x} \left[\frac{0}{0}\right]$$

Hence from L' Hospital Rule

$$= \lim_{x\to 0} \frac{1\times \sin x + x\cos x}{\cos x + 1\times\cos x - x\sin x}$$

$$= \lim_{x\to 0} \frac{\sin x + x\cos x}{\cos x + \cos x - x\sin x}$$

$$= \frac{0+0}{1+1-0} = \frac{0}{2} = 0 \text{ Ans.}$$

(10) Evaluate $\lim_{x\to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$.

Ans.→ $\lim_{x\to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right) \infty - \infty$

$$= \lim_{x\to 0} \left(\frac{\sin^2 x - x^2}{x^2 \sin^2 x}\right) \left[\frac{0}{0}\right]$$

Hence from L' Hospital's Rule, we have

$$= \lim_{x\to 0} \frac{2\sin x\cdot\cos x - 2x}{x^2 \times \sin^2 x \times \cos x + \sin^2 x \times 2x}$$

$$= \lim_{x\to 0} \frac{\sin 2x - 2x}{x^2 \times \sin 2x + 2x\cdot\sin^2 x} \left[\frac{0}{0}\right]$$

Hence from L' Hospital's Rule

$$= \lim_{x\to 0} \frac{\cos 2x \times 2 - 2}{x^2 \times \cos 2x \times 2 + \sin 2x \times 2x + 2[x \times 2\sin x \cdot \cos x + \sin^2 x \times 1]}$$

$\left[\frac{0}{0}\right]$

$$= \lim_{x\to 0} \frac{2[\cos 2x - 1]}{2x^2 \cos 2x + 2x \sin 2x + 2x \sin 2x + 2\sin^2 x}$$

$\left[\frac{0}{0}\right]$

$$= \lim_{x\to 0} \frac{2[-\sin 2x \times 2]}{2[x^2 \times -\sin 2x \times 2 + \cos 2x \times 2x] + 4[x \times \cos 2x \times 2 + \sin 2x \times 1] + 2 \times 2\sin x \cdot \cos x}$$

$$= \lim_{x\to 0} \frac{-4\sin 2x}{2[-2x^2 \sin 2x + 2x \cos 2x] + 4[2x\cos 2x + \sin 2x] + 2\sin 2x}$$

$\left[\frac{0}{0}\right]$

$$= \lim_{x\to 0} \frac{-2\sin 2x}{[-2x^2 \sin 2x + 2x\cos 2x + 4x\cos 2x + 2\sin 2x + \sin 2x}$$

$$= \lim_{x\to 0} \frac{-2\sin 2x}{-2x^2 \sin 2x + 6x \cdot \cos 2x + 3\sin 2x} \quad \left[\frac{0}{0}\right]$$

$$= \frac{-2 \times \cos 2x \times 2}{-2[x^2 \times \cos 2x \times 2 + \sin 2x \times 2x] + 6[x \times -\sin 2x \times 2 + \cos 2x] + 3 \times \cos 2x \times 2}$$

$$= \frac{-4}{[-2 \times 0] + 6(1) + 6} = \frac{-\cancel{4}}{\cancel{12}} = -\frac{1}{3} \text{ Ans.}$$